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Longevity à la mode

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Objective

We aim to use the established mathematical properties to **estimate** the modal age at death. Secondly, to **estimate** other longevity-related results around the mode (especially: the rate of aging).

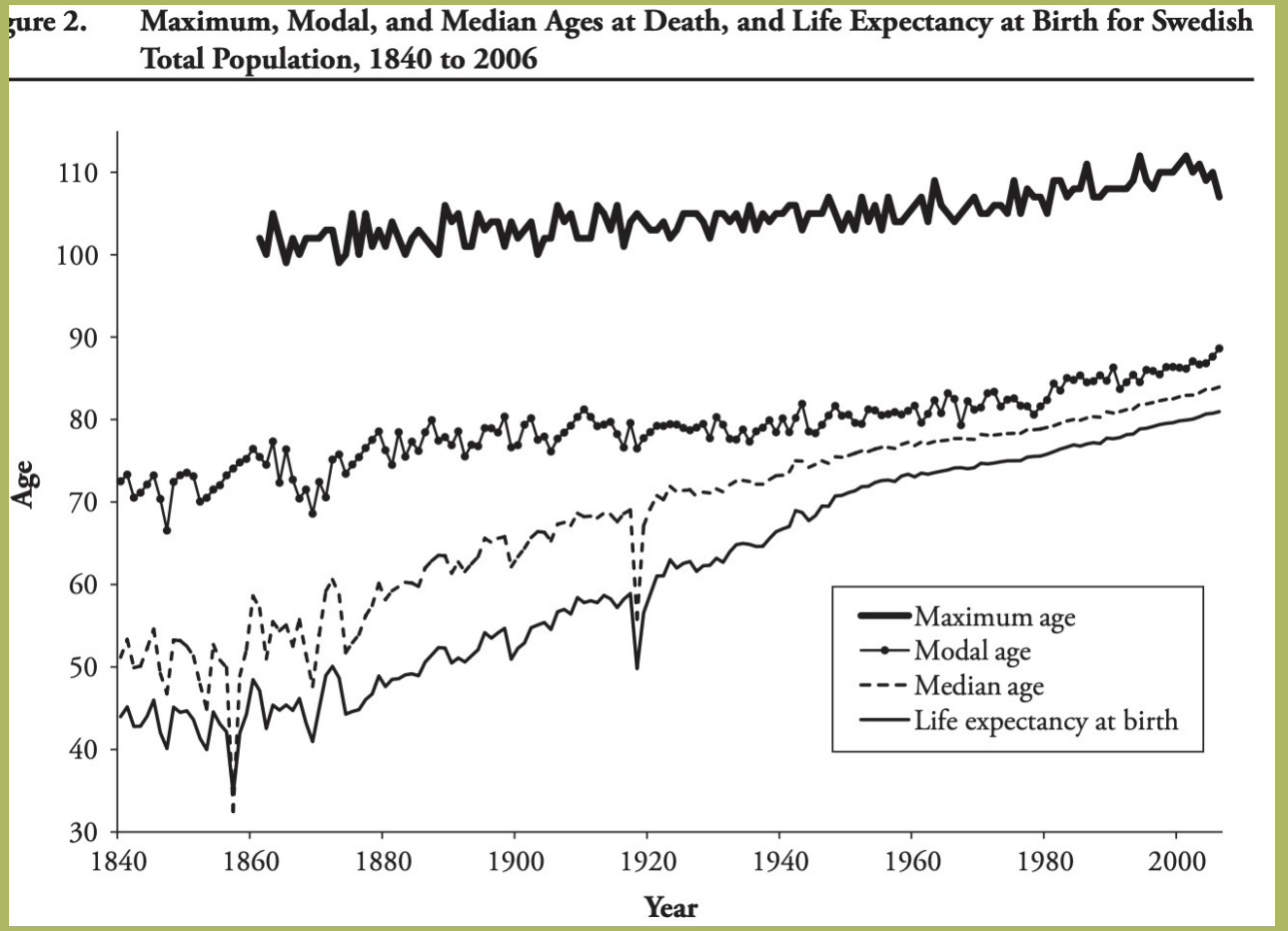
Introduction

Why the mode?

Why the mode?

A measure of longevity & dispersion:

i) Linear increase
ii) Maximum age and mode similar pattern
iii) is free from any old-age selection
iv) It can inform other phenomena, e.g. forecasting, compression,…
v) The individual is the same as the mode of the population ()

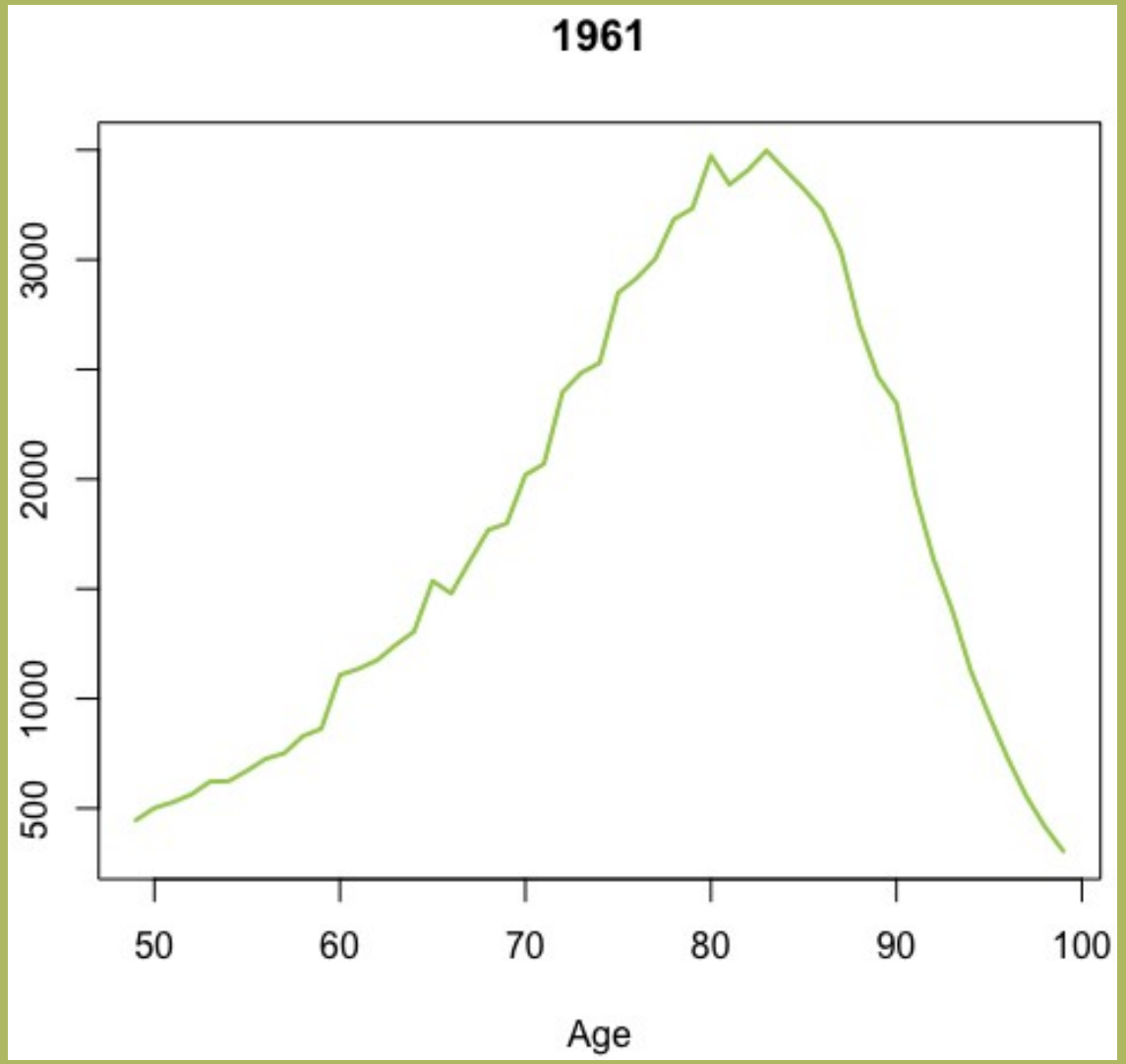


Canudas-Romo, V. (2010). Three measures of longevity: Time trends and record values. *Demography*

What is the problem with the mode?

What is the problem with the mode?

Estimation



What is the problem with the mode?

Estimation

Parametric Methods	Non-parametric methods
Models	Kannisto's method
	P-splines

What is the problem with the mode?

Models

Distribution		Modal age at death ()
Gompertz [1,2]		
Logistic [1]		
Gamma-Gompertz [2]		
Weibull [2]		

[1] Canudas-Romo, 2008
[2] Missov et al., 2015

What is the problem with the mode?

Kannisto's method

$$M_o = x + \frac{d(x) - d(x-1)}{(d(x) - d(x-1)) + (d(x) - d(x+1))}$$

What is the problem with the mode?

P-Splines method
(Ouellette & Bourbeau, 2011)

- Assume a Poisson distribution of death counts *d*
- Smoothed age pattern of mortality
- Smoothed age at death distribution

What do we know about the mode?

What do we know about the mode?

I) Definition

$$\frac{d}{dx} d\left(M_o\right)=0 \wedge ¿ \frac{d^2}{dx^2} d\left(M_o\right)<0$$

What do we know about the mode?

II) Property
(Canudas-Romo, 2008)

What do we know about the mode?

III) Property arithmetic

$$\frac{d\,\mu(M_o)}{dx} = \mu^2(M_o)$$

Introduction

Our method:

I) Definition

$$\frac{d}{dx} d(M_o) = 0 \wedge \frac{d^2}{dx^2} d(M_o) < 0$$

Our method:

II) Property
(Canudas-Romo, 2008)

$$ln\left(m\left(M_o\right)\right)-\ln\left(m\left(M_o-1\right)\right)=m\left(M_o\right)$$

Our method:

III) Property

$$\frac{\mu(M_o)}{dx} = \mu^2(M_o)$$

$$\mathbf{m}(M_o) - m(M_o - 1) = m^2(M_o)$$

Our method:

$$d(M_0) - d(M_0 - 1) = 0 \wedge ; d(M_0 + 1) - 2d(M_0) + d(M_0 - 1) < 0$$

$$¿ln(m(M_0)) - \ln(m(M_0 - 1)) - m(M_0) = 0$$

$$¿m(M_0) - m(M_0 - 1) - m^2(M_0) = 0$$

Our method:

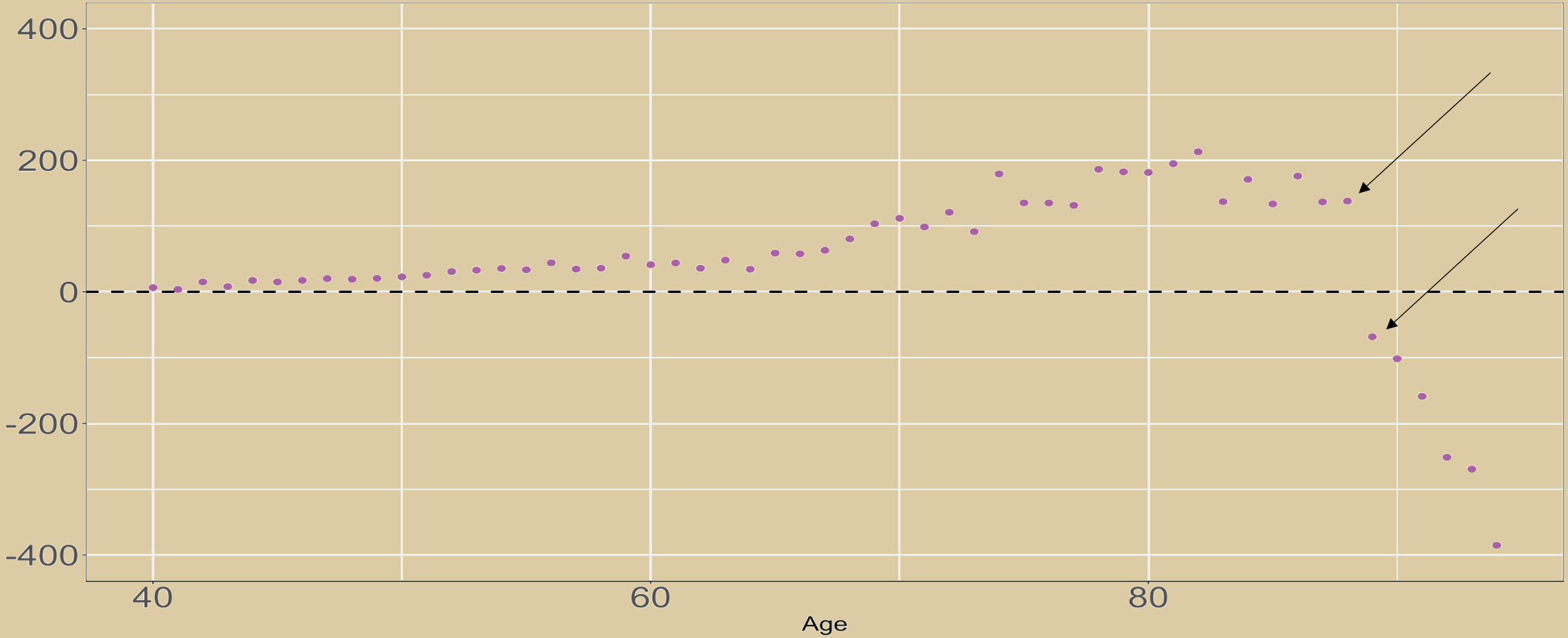


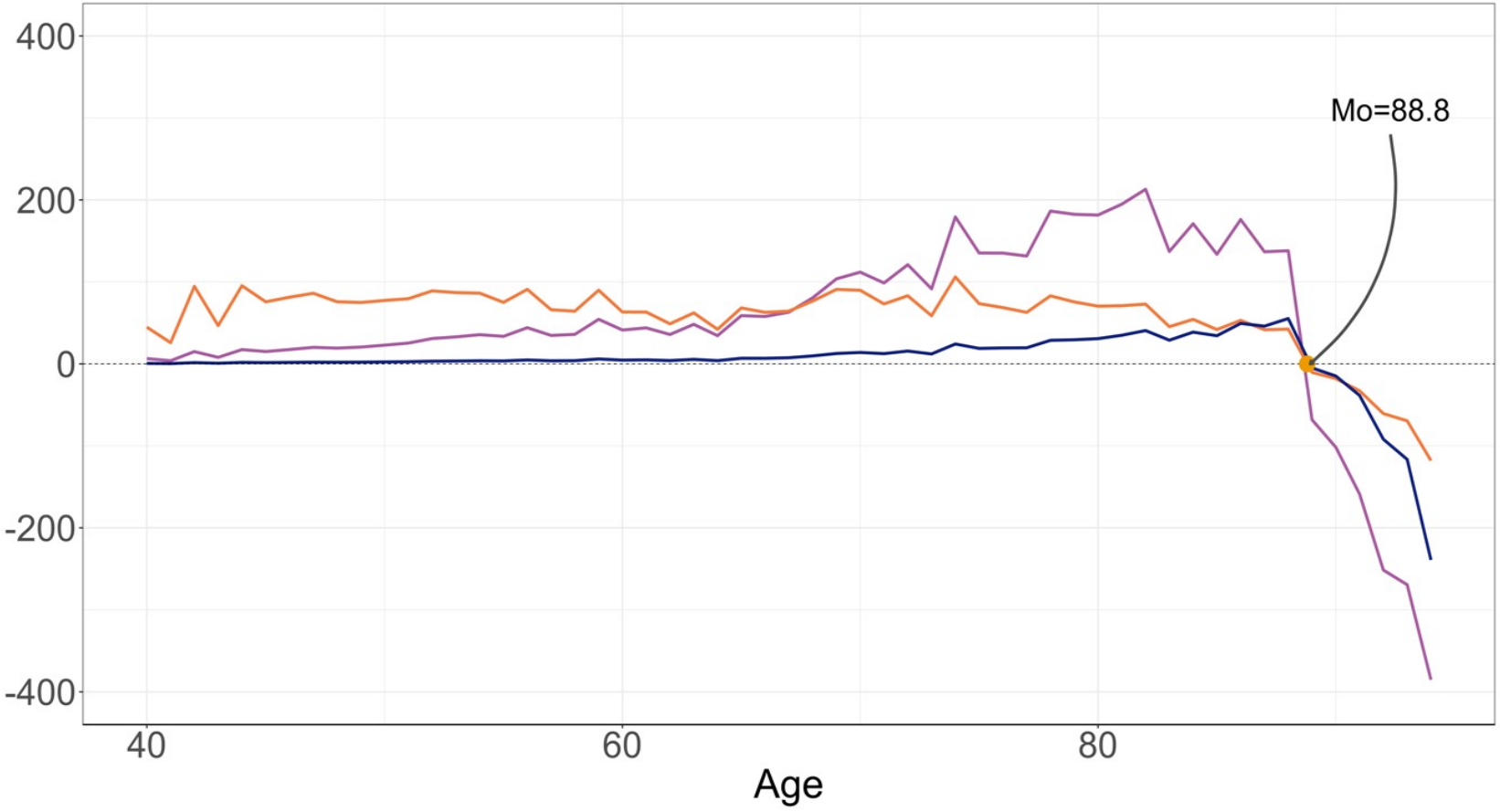
Illustration of the method

Data

- **Females** & Males
- Exposures & Death counts by age
- Six countries:
  - USA
  - France
  - Japan
  - Denmark
  - NDL
  - Spain

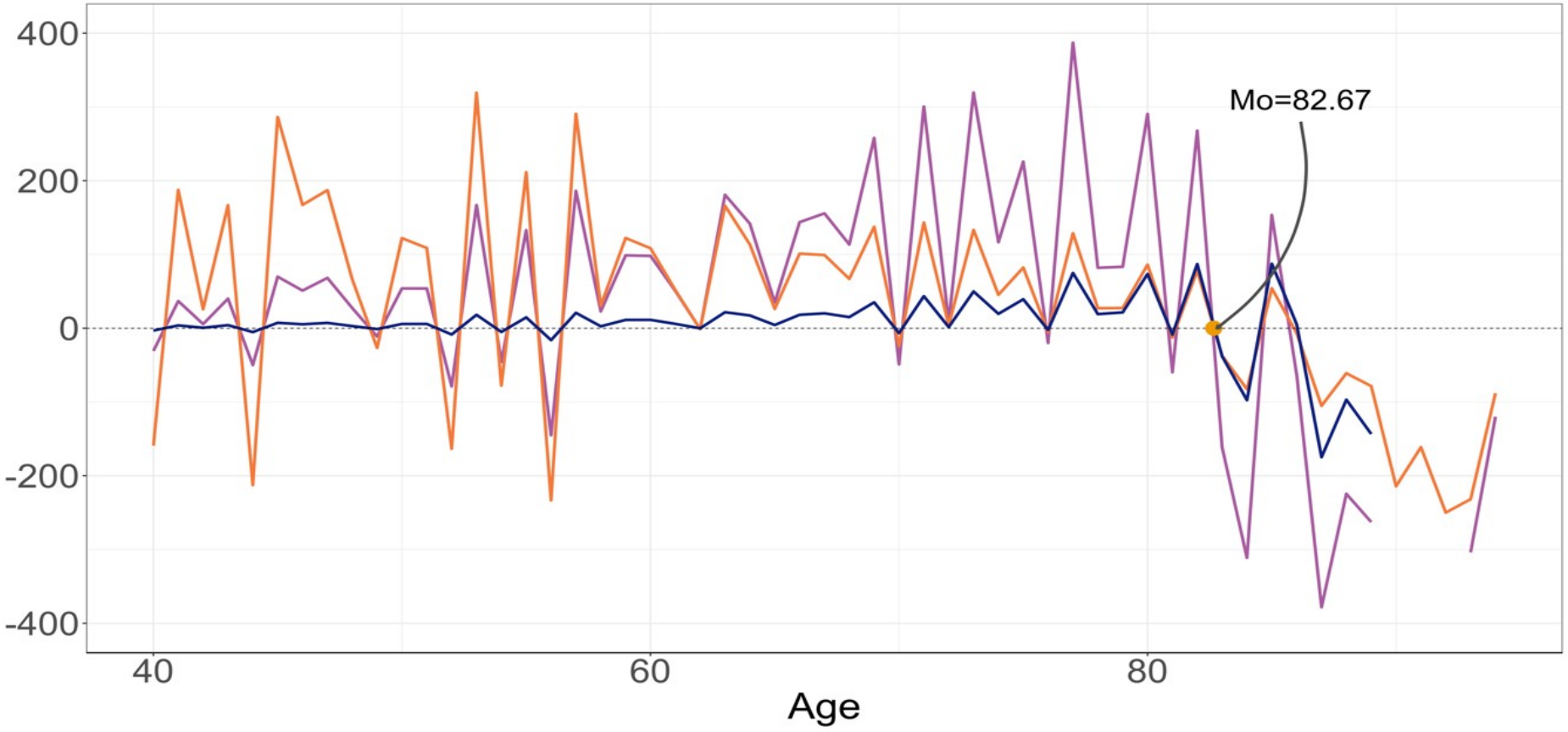
Case 1. Happy case: One crossing

Conditions by age and the modal age at death estimation based on its mathematical properties for females in 2019 in the USA

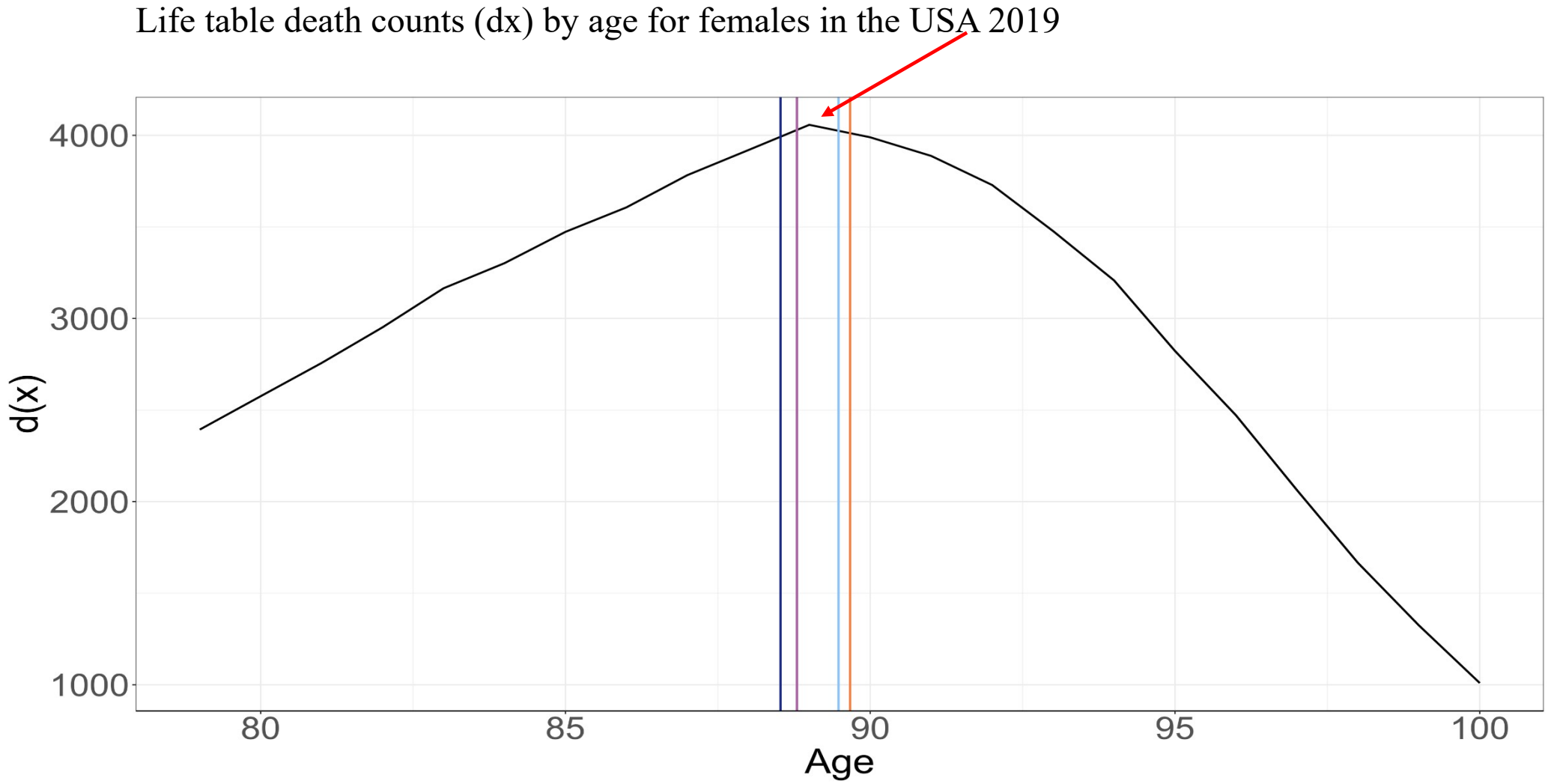


Case 2. (Less) Happy case: Multiple crossings

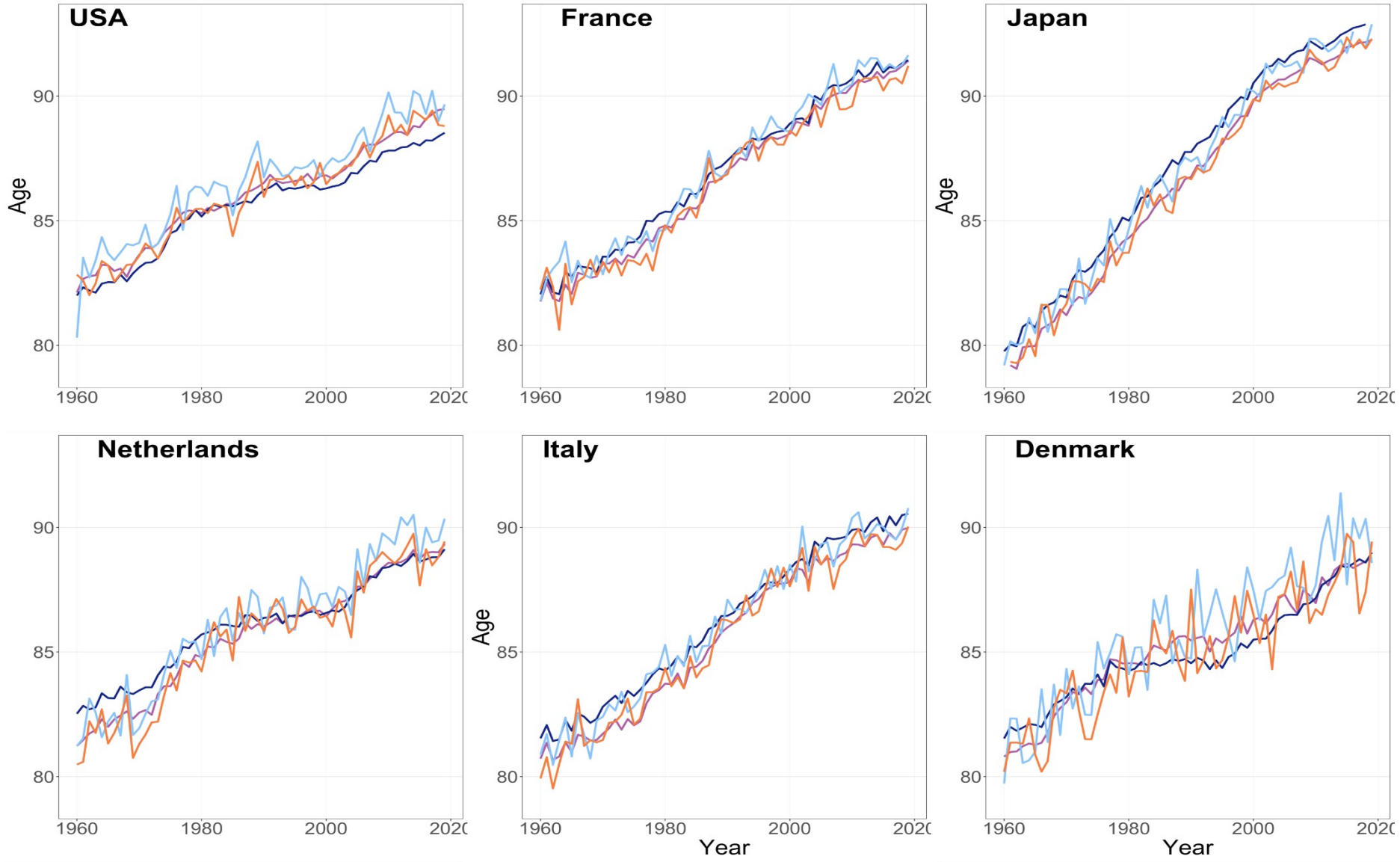
Conditions by age and the modal age at death estimation based on its mathematical properties for females in 1972 in Denmark



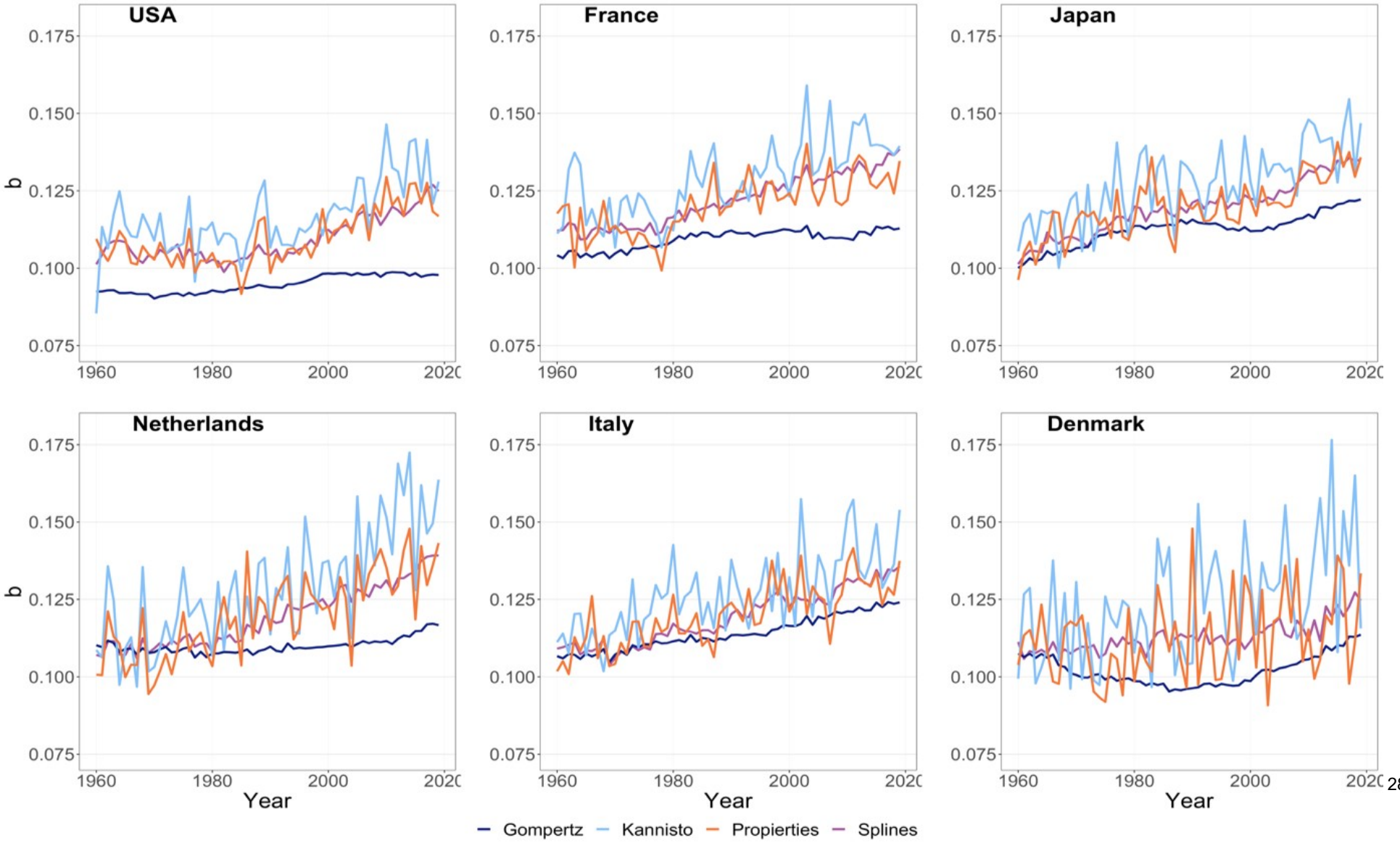
Comparison with other methods



Comparison with other methods



B-estimates


USA
France
Japan
Netherlands
Italy
Denmark
b
0.175
0.150
0.125
0.100
0.075
1960
1980
2000
2020
Year
Gompertz
Kannisto
Propierties
Splines

Discussion & Conclusion

The quest for better methods

- The mode as an indicator of lifespan
  - Interspecies (Horiuchi 2003)
- The mode as a tool to monitor longevity
  - Japanese females (Ouellette & Bourbeau, 2011)
- The mode as a forecasting tool
  - Basellini, Kjærgaard & Camarda, 2020
  - Bergeron-Boucher, Vazquez-Castillo & Missov, 2022 (pre-print)

Advantages

→ Easy to implement

→ No assumptions on the shape of mortality

→ No modelling

→ The estimated Mo fulfills the mathematical properties of Mo

→ Operates on discrete data

→ Detects and informs of changes in mortality

Limitations

→ The method is just to estimate the mode

→ More variability than other methods across years

Thank you!

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PREPRINT AVAILABLE

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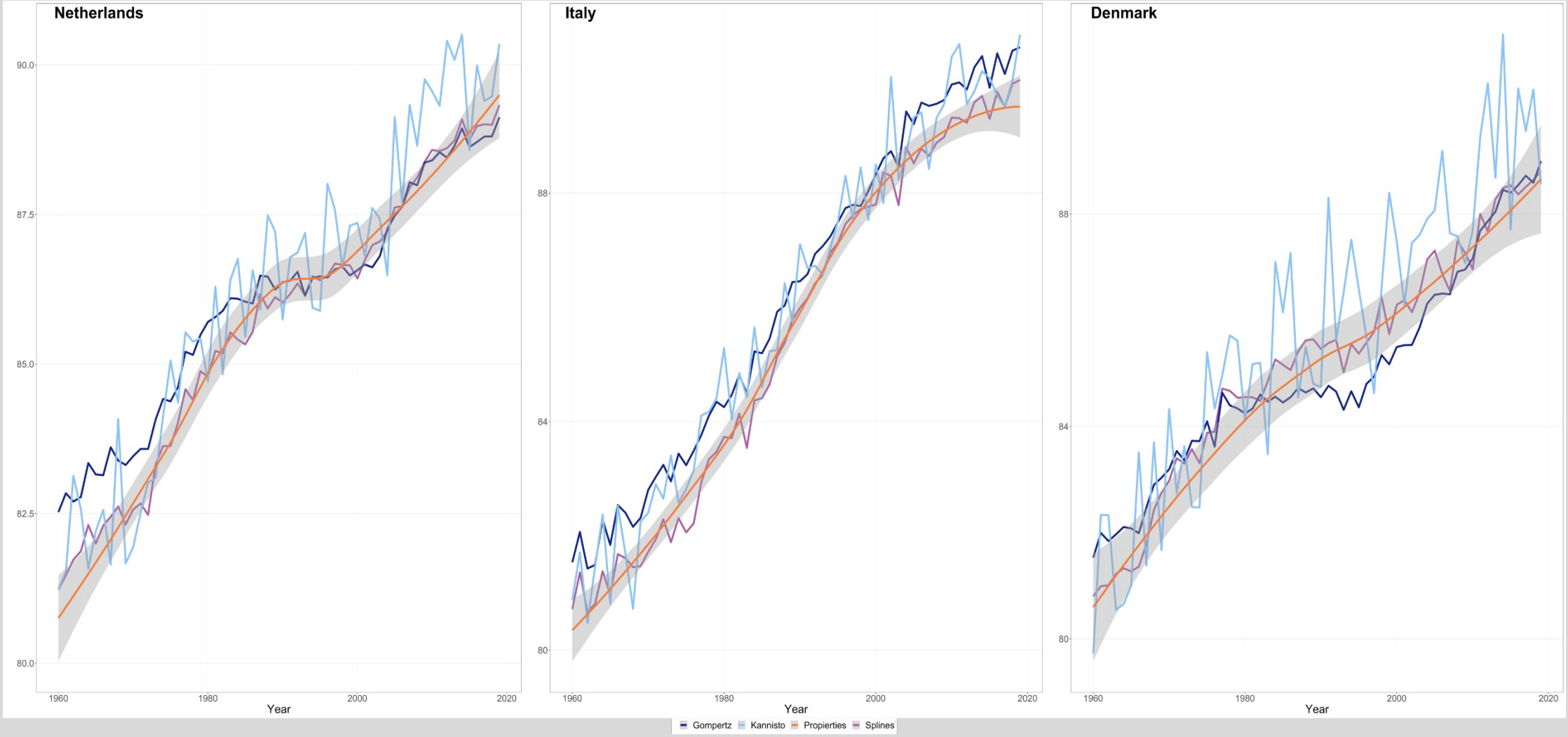
If we smoothed the properties mode…



Theory Parenthesis:

$$\frac{d}{dx} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

"When"

$$\approx \frac{f(x+1) - f(x)}{1} = f(x+1) - f(x)$$

Theory Parenthesis:

Preston, Heuveline & Guillot (2001)

In general:

$$\square_n m_x \approx \square_n M_x \approx \mu\left(x + \frac{1}{2}\right)$$