Projections of New Zealand Mortality Using the Lee-Carter Model and its Augmented Common Factor Extension

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Abstract
This paper presents the results from an empirical study on projecting New Zealand mortality. First, we analyse mortality data from 1948 to 2009 to obtain background information for the modelling process. In particular, we examine various aspects of mortality patterns by gender such as the differences across age, declining trend over time, old-age mortality rates, movement of the survival curve, modal age at death, and continual increase in life expectancy. We then apply the Lee-Carter model and its augmented common factor extension to the mortality data and project the death rates and life expectancy. We investigate the optimal starting year for fitting the model, and carry out out-of-sample and residual analyses to assess model performance. The fitted models appear to provide further insight into the underlying mortality trends. In addition, we make use of a hypothetical example of a pension to illustrate potential financial impact of longevity risk.

Life expectancy at birth in New Zealand has increased from 69 years in 1948 to 81 in 2009. For the past ten years, life expectancy has risen by 0.3 years per annum on average, and there is no sign of the upward trend ending in the foreseeable future. It is hence important for the government and insurance companies to properly allow for this trend and the so-called ‘longevity risk’ – the risk that a pension scheme or an insurer’s annuity portfolio pays out more than expected due to increasing life expectancy. Omission or miscalculation of this risk could potentially lead to disastrous financial outcomes.

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There are many ways to project future mortality rates. Booth (2006) provides a comprehensive review of demographic forecasting methodologies and mentions that there are broadly three approaches: extrapolation, expectation, and explanation. Some researchers criticise the extrapolation approach as being overly simple (e.g. Gutterman & Vanderhoof, 1998), in the sense that it implicitly assumes that past patterns would repeat in the future and does not take into account expert opinion or the structural relationship between the demographic and other variables. We believe, however, that studying the past patterns and trends carefully forms a solid basis for understanding how mortality changes over time, and projecting these past trends into the future serves as a robust first-step or benchmark for further analysis. Moreover, as noted in Lee and Miller (2001), there has historically been a pessimistic bias of expert opinions on mortality, which suggests that present knowledge reflects on current limitations but not future means of breaking through them. In practice, the distinction between different approaches is less clear-cut and one can always adjust the extrapolation procedure by expert opinions or related variables where appropriate. In this paper we attempt to recognise the underlying patterns of mortality changes and adopt the extrapolation approach to perform projection of future mortality and life expectancy.

We collected New Zealand deaths and population data by gender and single age for years 1948 to 2009 from Statistics New Zealand’s Infoshare series (www.stats.govt.nz) and the Human Mortality Database (www.mortality.org). Figure 1 illustrates the changes in the population size over time for the 1880, 1900, 1920, 1940, 1960, 1980, and 2000 cohorts. Without volatile migration movements and abrupt events, the curves are expected to progress smoothly for each year due mainly to the occurrence of deaths. While most parts of the curves are rather smooth, it is interesting to see a few irregular patterns, e.g. the small peak at mid-30s for the 1940 cohort and the severe trough at mid-20s for the 1960 cohort. These erratic patterns exist for a number of consecutive cohorts (not shown here) and could arise from certain temporal migration trends in the past.
Observed Mortality Patterns

Mortality Decline

Figure 2 plots the log death rates against age for years 1948, 1980, and 2009. The shapes of the curves are typical in nature: a drop from high infant mortality for early childhood, then a rise to the peak at around age 20 (more noticeable for males; the so-called accident hump), followed by a linear increase for adult mortality. While it is obvious to see that mortality generally declines over time, the decrease has been uneven across different ages and periods. In particular, the decline at middle to old ages is much more significant between 1980 and 2009 than between 1948 and 1980, especially for males. As mortality at young ages has already reached very low levels, improvement at older ages would tend to play a bigger role than previously for future increase in life expectancy. Figure 3 also demonstrates that in recent decades, mortality improvement has become more prominent at ages 50 and above. With continual breakthroughs in health care and medical technology, it is not unrealistic to expect this trend to last for the next few decades. In addition, we note that the 20-29 rates rise temporarily around 1975 and then get back to the declining trend in about 1985. This change in direction explains why in Figure 2 the accident hump is greater in 1980 but smaller in 2009.
Figure 2: Log death rate for years 1948, 1980, and 2009

Figure 3: Log death rate for age groups 0 to 80-89

Old-Age Mortality

The data collected span across ages 0 to 110+. As shown in Figure 4, the death rates are more volatile from age 90 onwards, especially for ages 100 and above, due to small exposures of these very old ages. Projecting the patterns of these ages into the future may produce unstable results. Hence we chose to exclude the data of ages 90 and above in our model fitting in the later sections.
Figure 4: Death rate, log death rate, and logit death rate for years 1948, 1980, and 2003

In order to ‘close out’ the life table for calculating life expectancy and deriving the survival curve, we apply the logistic model as proposed by Thatcher (1999). Compared to the traditional Gompertz, Weibull, and Heligman and Pollard models, the logistic model has been found to provide clearly closer fit to a large amount of old-age (80 years old and over) death data of around 13 countries over 30 years. The resulting death rate curve would have a point of inflection (generally around age 100 or above) and the log death rate curve would have a decreasing slope against age.
Deduced from the logistic model, Thatcher (1999) proposes the following linear relationship for high ages:

\[ \logit(\mu_x) = \ln \alpha + \beta x \quad (1) \]

in which \( \mu_x \) is the force of mortality at age \( x \), and \( \alpha \) and \( \beta \) are regression parameters that can readily be estimated. This linear relationship has been shown to furnish a good approximation to old-age mortality. This relationship was adopted to the death rates from ages 70 to 89 and the rates to ages 90 to 109 were projected. The fitted/projected rates (including the log and logit rates) were then plotted as curves in Figure 4.

It can be seen that the model fitting appears to be satisfactory before age 90 (the R² values for all 62 years of data of both genders have an average of 0.98 with a minimum of 0.95.) Although the actual rates are rather volatile for ages 90 and over, they are still roughly in line with the model projections. As such, we will continue to utilize this technique in the following analysis. (We have also tried projecting the rates to age 119 and realise that the corresponding impact on life expectancy calculation is negligible, simply because the number of survivors is very small at such high ages. We therefore limit the projections to age 109.) In addition, there is a point of inflection on each death rate curve, and the log death rate curves have a decreasing slope across age.

**Survival Curve**

It is widely documented that the survival curves of many countries have been undergoing a process of transformation called rectangularization (e.g. Cheung et al., 2005). As infant and premature mortality (before ageing-related mortality) declines, the starting part of the survival curve becomes more horizontal, and as there is an increasing concentration of deaths around the modal age at death, the later part of the curve tends to be more vertical. The resulting effect is that the curve moves towards a rectangular shape, hence the term rectangularization.

As demonstrated in Figure 5, such phenomenon has also been happening in New Zealand. For males, it can also be seen that the movement of the curve at older ages is much larger between 1980 and 2009 than between 1948 and 1980, because mortality decline at these ages is more
significant during the later period, as discussed above. Over recent decades, since infant and premature mortality has reduced to historically low levels, horizontalization of the survival curve has turned into a relatively insignificant process; and while verticalization appears to continue to take place, the movement of the curve for males in recent years develops into something slightly similar to a parallel shift to the right, for certain parts of the curve. We suspect that the latter phenomenon would be more prominent if both life expectancy and maximum lifespan keep on increasing in the future.

Figure 5: Survival curve for years 1948, 1980, and 2009

![Survival curve for years 1948, 1980, and 2009](image)

Figure 6 shows how the modal age at death, M, and the standard deviation of the ages at death above M, SD(M+), change over the period since 1980. As shown, there is a clear tendency for M to increase over time for both genders. Moreover, there is also a decreasing trend for SD(M+), which indicates an increasing concentration of deaths around M. (If smoothed values of M are used instead, the decreasing trend of SD(M+) would be clearer and less volatile.) The combined effect is enhancing verticalization of the survival curve, which is in accordance with our discussion above.
**Figure 6: Modal age at death and standard deviation of ages at death above modal age**

![Figure 6: Modal age at death and standard deviation of ages at death above modal age](image)

**Life Expectancy**

Figure 7 shows clearly that life expectancy at birth, as based on our computation, has been rising gradually from 71 for females and 67 for males in 1948 to 83 for females and 79 for males in 2009. The slope of the trend has increased sharply since around 1985, particularly for males. As discussed earlier, the change in the slope is due largely to mortality improvement at older ages, which is also reflected by the considerable change in the growth rate of life expectancy at age 65 during the same period.

**Figure 7: Life expectancy at birth and life expectancy at age 65**

![Figure 7: Life expectancy at birth and life expectancy at age 65](image)

The increasing trend seems to have slowed down slightly from the middle of 1990s. The upward trend, however, remains fairly persistent and there is no indication that it would come to an end in the foreseeable future. Though we cannot deny the possibility of unpredicted major structural shifts or catastrophic events, it seems rather likely that life expectancy at
birth, and especially at older ages, would continue to rise steadily for the
next few decades, under ongoing advancement in medical knowledge and
health care and continuing decline in smoking prevalence (e.g. Harper &
Howse, 2008).

**Preliminary Fitting of Lee-Carter Model**

The Lee-Carter model (1992) remains the most popular model for projecting
future mortality rates, with various extensions and modifications suggested
by different authors (e.g. Lee, 2000). The basic model has the following
form:

\[ \ln m_{x,t} = a_x + b_x k_t + \epsilon_{x,t} \]  (2)

in which \( m_{x,t} \) is the central death rate at age \( x \) in year \( t \), \( a_x \) depicts the
general mortality pattern, \( b_x \) measures the sensitivity of the log death rate
to changes in the mortality index, \( k_t \) is the mortality index, and \( \epsilon_{x,t} \) is the
Corresponding residual term. The \( a_x \) parameters are estimated by
averaging \( \ln m_{x,t} \) over time. The \( b_x \) and \( k_t \) parameters are then obtained
by applying singular value decomposition (SVD) to \( \{ \ln m_{x,t} - a_x \} \), subject
to two constraints \( \Sigma b_x = 1 \) and \( \Sigma k_t = 0 \). Finally the \( k_t \) parameters are re-
estimated so that the fitted number of deaths and the actual number of
deaths are equal for each year. The strengths of this model are mainly its
simplicity and also its ability to produce a highly linear series of \( k_t \) across
time for a number of countries’ data (e.g. Lee and Miller 2001). This
linearity allows one to simply use a random walk with drift to model the \( k_t \)
series and highly facilitates projection of mortality rates.

We apply the Lee-Carter model to our data from years 1948 to 2009.
The parameter estimates are set out in Figure 8. The \( a_x \) estimates, as
expected, have a typical shape of log death rates across age. Moreover, the
\( b_x \) estimates indicate that the highest sensitivity to general mortality
decline occurs at two age ranges: infant and early childhood, and middle age.
Similar patterns are found by other researchers, e.g. Booth et al. (2002).
Interestingly, for the $k_t$ estimates, the decreasing trend is not linear and there is a change in the slope in around 1985. Afterwards, the trend is very close to linearity, which implies that it may be more appropriate to choose a later starting year for the model fitting, in order to avoid the systematic change in the $k_t$ series and reduce the variability of projected rates. Nevertheless, it can also be argued that more sophisticated time series models such as ARIMA models can readily be adopted to deal with any non-linearity in the series while preserving the use of the full set of data.
Regarding this viewpoint, we would like to emphasise three important issues. Firstly, the observations made previously reveal that there have been some structural changes in mortality improvement since 1980s. Inclusion of the patterns before these changes would probably be undesirable for model projections. Secondly, one of the main reasons behind the Lee-Carter model’s popularity is the highly linear series the model generates based on the data of different countries. This linearity makes the analysis of mortality decline and its projection much more straightforward and sensible. Finally, as illustrated in Figure 9, we discover that the $b_X$ estimates using different starting years exhibit varying patterns. Although the $b_X$ estimates from different fitting periods are not directly comparable, it would still be preferable to select a fitting period not just as long as possible, but also within which the parameter estimates possess more consistent behaviour when switching from one starting year to the next. Booth et al. (2002) study Australian data and discuss the same issues of non-linearity of $k_t$ and invalidity of the assumption of constant $b_X$ over time.

We have tested each starting year from 1948 to 2000 and find that the parameter estimates display some extent of instability when the fitting period is reduced to less than 20 years or so. Accordingly, we will use at least 15 years of data in the following analysis and model fitting. Figure 9 depicts two distinct patterns of the $b_X$ estimates obtained from starting before and after 1980 for both genders. The main difference is that for the later starting years, there is a peak at around age 25, in addition to those at early childhood and middle age. It is also observed that the $b_X$ estimates are less stable when the fitting period is shortened in the latter cases, and this reduction in parameter robustness is a cost of using only the most recent and relevant data.
Additionally, Figure 10 sets forth the R2 and adjusted R2 values of testing linearity of the $k_t$ series for different starting years. For both females and males, the R2 values reach their maximum with year 1985, being 0.98 and 0.99 respectively. This result is in line with how $k_t$ changes in its slope in Figure 8. To sum up, the observations thus far have provided much support for choosing a later starting year, while keeping a fitting period of at least 15 years. Based on all the analysis above, we decide to use 1985 as the starting year, instead of the whole data set, for the rest of this paper. The resulting $k_t$ series are found to be highly linear in nature.

In the projections below, the observed death rates in the last year of the fitting period are taken as the starting point of projection and $k_t$ is constrained to start at zero in the projection period, as suggested in Lee and Miller (2001).
For further evaluation of model performance, the Lee-Carter model is now fitted to two periods 1985-1999 and 1985-2004. The projected death rates and life expectancy are compared against the observed values for the rest of the period ending in 2009. In particular, the mean absolute percentage error (MAPE), the projected path of life expectancy, and the residuals are examined.

Table 1 lists the MAPE of the projected log death rates for the two fitting periods. The MAPE for each case is around 3% to 4%, and the model prediction appears to be satisfactory overall. The differences between the two fitting periods are quite small, indicating that as long as the linearity of $k_t$ continues and is properly accounted for, decent prediction accuracy could be achieved, with less reliance on the fitting period. Furthermore, the MAPE of the fitted log death rates for each case is close to 2%, and so the model fits reasonably well.

Table 1: MAPE of projected log death rates

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<tbody>
<tr>
<td>Females</td>
<td>3.9%</td>
<td>3.6%</td>
<td>3.5%</td>
</tr>
<tr>
<td>Males</td>
<td>3.2%</td>
<td>3.1%</td>
<td>3.2%</td>
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Figure 11 plots the projected life expectancy at birth with the 95% confidence intervals (dotted lines) against the observed values (solid lines). The intervals are constructed taking only the variability of $k_t$ into account, but not that of $\varepsilon_{x,t}$ and the standard error of the parameters. As shown, there is some underestimation in the projected values, the size of which is around 0.2 to 0.3 years on average. Nevertheless, the performance still looks reasonably good since the underestimation is rather mild and all the observed values lie well within the confidence bounds. (If the fitting period is 1948–1999 instead, the underestimation is 0.9 years on average and tends to increase with the length of projection.) Lee and Miller (2001) also demonstrate under-projected gains with the Lee-Carter model, but to a lesser extent than government forecasts.

**Figure 11: Projected life expectancy at birth for fitting periods 1985–1999 and 1985–2004**
Ideally, the residuals from the fitted model would be randomly spread with no such features as trends or clusterings. For example, if there are unusual patterns when the residuals are plotted against the year of birth, it may be necessary to add a term to (2) to allow for the cohort effect. Figure 12 shows the positive and negative residuals (age vs calendar year) in white and black cells respectively. The signs of the residuals are fairly randomly distributed and there does not seem to be any systematic effect left over in the residuals. Moreover, there are no particular patterns along the diagonal direction, and so allowance for the cohort effect is not needed. The model structure can thus be regarded as being adequate in capturing the main attributes of the data, namely the age effect and period effect.

**Figure 12: Residuals for fitting periods 1985-1999 and 1985-2004**


In spite of the satisfactory performance discussed above, separate projections of female and male mortality lead to two potential problems. The first is that the ratio of death rates between genders may or may not change significantly over time, and modeling two genders separately does not take this ratio into consideration at all. Figure 13 compares the projected ratio in 2009 against the observed ratio for 10-year age groups. The model projections clearly fail to realise the change in the ratio at younger ages, especially at 20-29, though the outcome at older ages is much better, due to the stability of the ratio for those ages. As illustrated in Figure 14, historically this ratio is more volatile at younger ages than at older ages, which explains why the projected ratio is more out of line for the former. In addition, except for age groups 0 and 30-49, there is an interesting pattern generally, in which the ratio increases between 1948 and 1980 and then decreases between 1980 and 2009. It is difficult to predict whether this decline in the ratio would continue in the future, and if so for how long. If the focus of the analysis is on pricing annuities for an insurer or valuing pension schemes for the government, however, failing to incorporate the gender mortality ratio into the model is unlikely to cause a serious problem, as long as the ratio remains relatively stable at older ages and the projection period is not too long.
There is actually another problem hidden in the modelling process as the projection period is only 5 to 10 years. This problem can turn out to be much more severe when the projection period is longer. As detailed in the appendix of Li and Lee (2005), if each gender is estimated with its own $k_i$ series which is then extrapolated linearly and independently from the other gender, the female and male projected death rates would diverge progressively over time. This divergence is purely a modelling artifact and may need to be avoided in certain applications. One way to reduce this divergence is to adopt the augmented common factor extension of the Lee-Carter model, as discussed below.
Augmented Common Factor Lee-Carter Model

The augmented common factor Lee-Carter model is proposed in Li and Lee (2005) as:

$$\ln m_{x,t,i} = a_{x,i} + B_x K_t + b_{x,i} k_{t,i} + \epsilon_{x,t,i} \quad (3)$$

in which the meanings of the expressions here are similar to those in (2), except that the subscript $i$ indicates the gender considered, and that the additional terms $B_x$ and $K_t$ refer to the aggregate population. In brief, $B_x K_t$ (common factor) allows for the main trend in mortality change of the whole population and $b_{x,i} k_{t,i}$ (specific factor) represents the short-term difference from the main trend for gender $i$. The $B_x$ and $K_t$ parameters are first obtained from applying the original Lee-Carter model (i.e. via $b_x$ and $k_t$ in (2)) to the aggregate data combining two genders. All the other parameters are then estimated separately for each gender, by fitting (2) to $\{\ln m_{x,t,i} - B_x K_t\}$. The $K_t$ series is modelled as a random walk with drift as previously. In order to accommodate the short-term difference from the common factor and at the same time ensure this difference fades out in the long run, the $k_{t,i}$ series for females and males are modelled by an AR(1) model. In this way, $k_{t,i}$ converges to a constant value as the length of projection increases and the disparity from the common factor then gradually disappears. Eventually, the projected ratio of male to female death rates turns into a constant at each age. The choice of the AR(1) model is also driven by its straightforwardness in fitting and implementation. We find below that for both genders the autocorrelations of the residuals are statistically insignificant for the first 6 lags (except lag 5), and there is no reason to complicate the analysis of a limited amount of data with a more sophisticated (e.g. higher order) but largely unjustifiable model.

We now apply the Lee-Carter model and also the augmented common factor extension to the period from 1985 to 2009. As listed in Table 2, the MAPE of the fitted log death rates for each case is slightly more than 2%, with little difference between the two models. Conventionally, it is more preferable to have the maximum length of the projection period
approximately equal to the length of the fitting period. Thus we project the
death rates and life expectancy only up to 2040. Figure 15 demonstrates
that the projected values of life expectancy from the two models are fairly
close. For example, life expectancy at birth for females in 2040 is projected
to be 88.6 and 88.8 respectively by the two models, and for males 86.4 and
85.9. The augmented common factor model produces a higher value of life
expectancy for females but a lower value for males. The main reason is that
the drift term of the random walk of $K_t$ (-2.38), being identical for both
genders, is roughly equal to the average of those of $k_t$ obtained from
applying the Lee-Carter model separately for females (-2.22) and males (-
2.59). As revealed in Figure 16, the projected values depend heavily on the
drift term of the random walk, and so there is a question on whether it is
suitable to employ a common mortality index for both genders as in the
augmented common factor model or to deal with each gender independently.

Table 2: MAPE of fitted log death rates

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<tr>
<th>Gender</th>
<th>Lee-Carter</th>
<th>Augmented Common Factor</th>
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<tbody>
<tr>
<td>Females</td>
<td>2.3%</td>
<td>2.3%</td>
</tr>
<tr>
<td>Males</td>
<td>2.1%</td>
<td>2.1%</td>
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Figure 15: Projected life expectancy at birth from Lee-Carter model and augmented common factor model
Figure 16: Projected life expectancy at birth in 2020, 2030, and 2040 vs drift term (in magnitude) of $k_t$ for Lee-Carter model and $K_i$ for augmented common factor model

The answer to this question probably depends on the purpose of the analysis. Firstly, it is noted that the differences in projected life expectancy are small between the two models, even the length of projection is 31 years. If the female and male projected death rates are further aggregated in some way, the disparity amongst the two models would be minimal, due to the offsetting effect as noted in the example above. Hence the choice of which model to use seems to be not much of an issue if one intends to focus on projecting life expectancy. On the other hand, if the ratio of death rates between genders is a matter of concern, such as in population projection and policy planning, it would be more sensible to avoid the divergence problem arising from separate projections of female and male mortality, as discussed previously. In this regard, the augmented common factor model would make
sure the projected ratio of male to female death rates at each age converges to a constant. All these issues are clearly reflected in Figure 17. It can be seen that the gender mortality ratios converge under the augmented common factor model. In contrast, the ratios move in various directions under the initial Lee-Carter model.

Figure 17: Projected ratio (male to female) of death rates for different years and age groups from Lee-Carter model and augmented common factor model
Because the estimated dependency of $k_{ij}$ across time is quite weak and the observed death rates in the final year of the fitting period are treated as the starting point of projection, the convergence is achieved quickly and the overall 2009 patterns remain in the projections. Some possible modifications are: averaging the observed rates in the last few years of the fitting period for the starting point of projection; and arbitrarily choosing parameters or model structures which bring about stronger time dependence.

Figure 18 plots the observed log death rates against age for years 1948, 1980, and 2009, and the projected rates in 2040 from the augmented common factor model. The typical shapes in the past are broadly preserved in the projection, but there are two anomalies. First, mortality improvement at around ages 30–44 has been relatively slow compared to other ages for recent decades, so the projected curve at this age range bulges up slightly. On the other hand, mortality decline at ages 50–79 has been faster than at adjacent ages, causing the projected curve less linear than the historical ones for adult mortality. If one is doubtful about whether these extrapolated features would really emerge and believes the typical shapes should be maintained, certain parameterized curves could be fitted or arbitrary adjustments be made to the projected rates.

Furthermore, the projected survival curve in Figure 19 reveals that there is continual verticalization in the projection. With regard to closing out the life table and plotting the survival curve, there is some limitation here as the death rates are only extended to age 109 using (1). More precise and voluminous old-age data are needed to verify the suitability of this technique in the case of a long projection period. As stated earlier, we suspect that there could be some degree of a parallel shift of the curve to the right under enhancing life expectancy and maximum lifespan in the future, and the application of (1) may need to be modified.
Figure 18: Observed and projected log death rates

Figure 19: Observed and projected survival curves
**Longevity Risk**

Finally, we construct a hypothetical example of a pension to illustrate potential financial impact of longevity risk, which is the risk that a pension scheme or an insurer’s annuity portfolio pays out more than expected because of rising life expectancy. The following results imply that miscalculation or omission of this risk could lead to undesirable financial consequences.

Consider a pension with regular payments of $1 payable in arrear to a female aged exactly 65 on 1 January 2010. The discount rate is assumed to be 6% p.a. (The New Zealand 10-year government bond yield was around 6% in January 2010.) We simulate 1,000 paths of $k_t$ from 2010 onwards, based on the results of applying the Lee-Carter model to the period from 1985 to 2009 in the previous section. Both the variability of $k_t$ and the standard error of the drift term are taken into account, while the variability of $\varepsilon_{x,t}$ is not included. As the latter risk can be diversified by having more independent lives in a pension scheme, we focus on the former risk which cannot be diversified in the same way. The simulated $k_t$ values are then used to generate 1,000 samples of the present value of the pension.

Using the 2009 death rates without allowing for mortality improvement, the expected present value of the pension is $10.97 as at 1 January 2010. Figure 20 shows that this value falls into the lower 1 percent region of the sampled distribution of the present value of the pension, which has a sample mean of $11.53 and 99th percentile of $12.10. In other words, completely ignoring mortality improvement would cause an underestimation of the reserves required to cover the pension by 5 percent in the sense of expectation and by 9 percent for the worst 1 percent scenario. The overall situation could get worse with the rapidly ageing population or if there are major changes in $k_t$, like the one in 1980s. From this simple example, we can see that proper allowance for longevity risk is of critical importance in maintaining financial stability for pension and annuity providers.
Concluding Remarks

This paper presents the results from an empirical study on New Zealand mortality for the period from 1948 to 2009. We started with analysis of various mortality patterns, and noticed a general mortality decline over the period, rectangularization of the survival curve, and steadily increasing life expectancy - in particular, structural shifts in the mortality trends. We then applied the Lee-Carter model and its augmented common factor extension to the mortality data. 1985 was identified as relatively suitable starting year for fitting the model. The following out-of-sample and residual analyses indicate satisfactory performance of the fitted models. It is noted that the initial Lee-Carter model and the augmented common factor model perform similarly on the whole, but the latter provides an edge in ensuring the convergence of the ratio of death rates between genders. Lastly, we showed through a pension example that failure to accommodate longevity risk could cause significant financial instability to pension and annuity providers.

As noted earlier, projecting relevant past trends into the future via an extrapolative approach can be regarded as a robust starting point. If the projection period is too long, however, this practice would be somewhat unrealistic, as it is difficult to justify the continuance of the same trend without major structural changes over such an extended period of time. When one takes the extrapolation method to perform projection, proper judgement regarding the fitting period, projection period, parameter values, and reasonability of final results should be exercised.
References


Human Mortality Database. (2010). University of California, Berkeley (USA) and Max Planck Institute for Demographic Research (Germany). www.mortality.org.


